Lightning Round: Set 1

1. What is the value of $(2^{-1})^{-2}$?
2. Let $ABCDEF$ be a regular hexagon with side length 2. Calculate the area of $ABDE$.
3. Daniel can hack a finite cylindrical log into 3 pieces in 6 minutes. How long would it take him to cut it into 9 pieces, assuming each cut takes Daniel the same amount of time?
4. How many digits are in the base 10 representation of 3^{30} given $\log 3 = 0.47712$?

Lightning Round: Set 2

1. What is the sum of the first 10 primes?
2. Find all real x that satisfy the equation
$$\frac{1}{x+1} + \frac{1}{x+2} = \frac{1}{x}$$
3. Find the last digit of the number
$$\frac{400!}{(200!)(2^{200})}$$
4. The polynomial $x^3 - kx^2 + 20x - 15$ has 3 roots, one of which is known to be 3. Compute the greatest possible sum of the other two roots.

Lightning Round: Set 3

1. Bhairav the Bat lives next to a town where 12.5% of the inhabitants have Type AB blood. When Bhairav the Bat leaves his cave at night to suck of the inhabitants blood, chooses individuals at random until he bites one with type AB blood, after which he stops. What is the expected value of the number of individuals Bhairav the Bat will bite in any given night?
2. In triangle ABC, points M, N, and P lie on sides $\overline{AC}$, $\overline{AB}$, and $\overline{BC}$, respectively. If $\angle ABC = 42°$, $\angle MAN = 91°$, and $\angle NMA = 47°$, compute $\frac{CB}{BP}$.
3. Michael the Mouse stands in a circle with 11 other mice. Eshaan the Elephant walks around the circle, squashing every other non-squashed mouse he comes across. If it takes Eshaan 1 minute (60 seconds) to complete one circle and he walks at a constant rate, what is the maximum length of time in seconds from when the first mouse is squashed that Michael can survive?
4. Compute the greatest constant K such that for all positive real numbers a, b, c, d measuring the sides of a cyclic quadrilateral we have
$$\left(\frac{1}{a+b+c-d} + \frac{1}{a+b-c+d} + \frac{1}{a-b+c+d} + \frac{1}{-a+b+c+d}\right)(a+b+c+d) \geq K$$

Lightning Round: Set 4

1. How many divisors of 12! are perfect squares?
2. Let $ABCD$ be a square of side length 1, and let E and F be points on BC and DC such that $\angle EAF = 30$ and $CE = CF$. Determine the length of BD
3. Andrew the ant starts at vertex A of square ABCD. Each time he moves he chooses the clockwise vertex with probability 2/3 and the counter-clockwise vertex with probability 1/3. What is the probability that he ends up on vertex A after 6 moves?
4. Sierpinski's triangle is formed by taking a triangle, and drawing an upside down triangle inside each upright triangle that appears. A snake sees the fractal, but decides that the triangles need circles inside them. Therefore, she draws a circle inscribed in every upside down triangle she sees (assume that the snake can do an infinite amount of work). If the original triangle had side length 1, what is the total area of all the individual circles?

Lightning Round: Set 5

1. What is the integer closest to π^{π}? (No calculator allowed!)
2. If a train carrying 27 passengers leaves Grand Central Station at 8:00 AM and travels 900 miles due west to Chicago, arriving at 5:00 PM, what is the average speed of the train in miles per hour?
3. Alice the ant starts at vertex A of reglar hexagon ABCDEF and moves either right or left each move with equal probability. After 35 moves, what is the probability that she is on either vertex A or C?
4. Submit a positive integer x between 1 and 10 inclusive. Your score on the problem will be proportional to
$$\frac{11-x}{n}$$
where n is the number of teams that also submit the number x.