# 2017 Math Bonanza Team Round

CCA Math Team

January 14, 2017

Time limit: 40 minutes

T1) Given that $9 \times 10 \times 11 \times \cdots \times 15 = 32432400$, what is $1 \times 3 \times 5 \times \cdots \times 15$?

T2) A square of side length s is inscribed in circle C_1 and circumscribed about circle C_2. The area of the region in C_1 but outside C_2 is 25π. What is s?

T3) The operation $*$ is defined by $a * b = a + b + ab$, where a and b are real numbers. Find the value of

$$\frac{1}{2} * \left(\frac{1}{3} * \left(\cdots * \left(\frac{1}{9} * \left(\frac{1}{10} * \frac{1}{11} \right) \right) \right) \right).$$

T4) The 12 islands of the Bonanza archipelago are labeled $A, B, C, \ldots, K, L$. Some of the islands are connected by bridges, as indicated in the diagram below. Tristan wants be able to walk from island to island crossing each bridge exactly once (he doesn't care if he visits a given island more than once, or whether he starts and ends on the same island). Submit a pair of unconnected islands such that if they are connected by a bridge, Tristan can accomplish his goal.
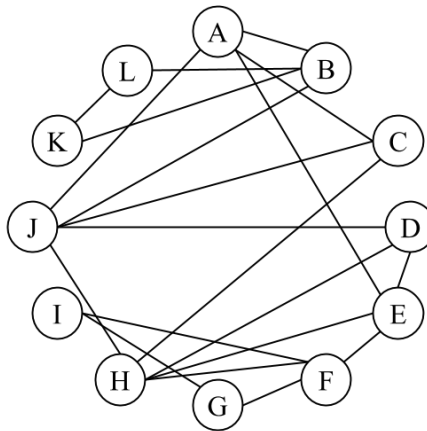
T5) Twelve people go to a party. First, everybody with no friends at the party leave. Then, after the i-th hour, everybody with exactly i friends left at the party leave. After the eleventh hour, what is maximum number of people left? Note that friendship is mutual.

T6) Alice and Bob take a and b candies respectively, where $0 \leq a, b \leq 3$, from a pile of 6 identical candies. They draw the candies one at a time, but one person may draw multiple candies in a row. For example, if $a = 2$ and $b = 3$, a possible order of drawing could be Alice, Bob, Bob, Alice, Bob. In how many ways (considering order of drawing and values of a and b) can this happen?

T7) Let $ABCD$ be a convex quadrilateral with $AC = 20$, $BC = 12$ and $BD = 17$. If $\angle CAB = 80°$ and $\angle DBA = 70°$, then find the area of $ABCD$.

T8) A group of 25 CCA students decide they want to go to Disneyland, which is 105 miles away. To save some time, they rent a bus with capacity 10 people which can travel up to 60 miles per hour. On the other hand, a student will run up to 9 miles per hour. However, because a complicated plan of getting on and off the bus may be confusing to some students, a student may only board the bus once. What is the least number of minutes it will take for all students to reach Disneyland?

Note: both the bus and students may travel backwards.

T9) Aida made three cubes with positive integer side lengths a, b, c. They were too small for her, so she divided them into unit cubes and attempted to construct a cube of side $a + b + c$. Unfortunately, she was 648 blocks off. How many possibilities of the ordered triple (a, b, c) are there?

T10) Triangle ABC is acute. Equilateral triangles ABC', $AB'C$, $A'BC$ are constructed externally to ABC. Let BB' and CC' intersect at F. Let CC' intersect AB at C_1 and AA' intersect BC at A_1, and let A_1C_1 intersect AC at D. If $A'F = 23$, $CF = 13$, and $DF = 24$, find BD.