2017 Math Bonanza Tiebreaker Round

CCA Math Team

January 14, 2017

TB1) Compute
$$12^3 + 4 \times 56 + 7 \times 8 + 9.$$

TB2) Let ABC be a triangle. D and E are points on line segments BC and AC, respectively, such that $AD = 60$, $BD = 189$, $CD = 36$, $AE = 40$, and $CE = 50$. What is $AB + DE$?

TB3) Let $\theta = \frac{2\pi}{2015}$, and suppose the product

$$\prod_{k=0}^{1439} \left(\cos(2^k \theta) - \frac{1}{2} \right)$$

can be expressed in the form $\frac{b}{2^a}$, where a is a non-negative integer and b is an odd integer (not necessarily positive). Find $a + b$.

TB4) Mr. Vader gave out a multiple choice test, and every question had an answer that was one of A, B, or C. After the test, he curved the test so that everybody got $+50$ (so a person who got $x\%$ right would get a score of $x + 50$). In the class, a score in the range $[90, \infty)$ gets an A, a score in the range $[80, 90)$ gets a B, and a score in the range $[70, 80)$ gets a C. After the curve, Mr. Vader makes this statement: "Guess A, get an A. Guess B, get a B. Guess C, get a C." That is, answering every question with the answer choice X would give, with the curve, a score receiving a grade of X, where X is one of A, B, C. Luke, a student in Mr. Vader's class, was told ahead of time that there were either 5 or 6 answers as A on the test. Find the sum of all possible values of the number of questions on the test, given this information.