2016 Math Bonanza Team Round

CCA Math Team

May 28, 2016

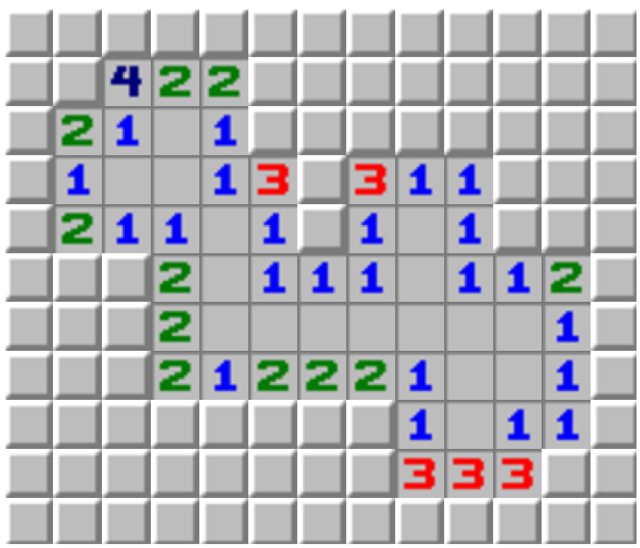
Time Limit: 40 minutes

T1) It takes 3 rabbits 5 hours to dig 9 holes. It takes 5 beavers 36 minutes to build 2 dams. At this rate, how many more minutes does it take 1 rabbit to dig 1 hole than it takes 1 beaver to build 1 dam?

T2) Perry the painter wants to paint his floor, but he decides to leave a 1 foot border along the edges. After painting his floor, Perry notices that the area of the painted region is the same as the area of the unpainted region. Perry's floor measures a x b feet, where $a > b$ and both a and b are positive integers. Find all possible ordered pairs (a, b).

T3) Find the sum of all integers n not less than 3 such that the measure, in degrees, of an interior angle of a regular n-gon is an integer.

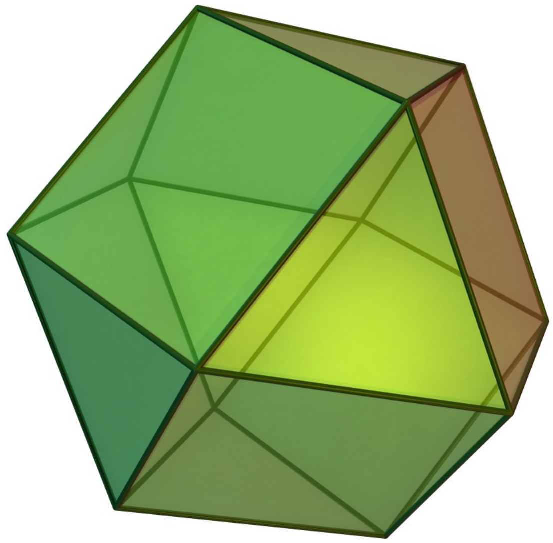
T4) In the *minesweeper* game below, each unopened square (for example, the one in the top left corner) is either empty or contains a mine. The other squares are empty and display the number of mines in the neighboring 8 squares (if this is 0, the square is unmarked). What is the minimum possible number of mines present on the field?



T5) How many permutations of the word "ACADEMY" have that there exist two vowels that are separated by an odd distance? For example, the X and Y in XAY are separated by an even distance, while the X and Y in XABY are separated by an odd distance. Note: the vowels are A, E, I, O, and U. Y is **NOT** a vowel.

T6) Consider the polynomials $P(x) = 16x^4 + 40x^3 + 41x^2 + 20x + 16$ and $Q(x) = 4x^2 + 5x + 2$. If a is a real number, what is the smallest possible value of $\frac{P(a)}{Q(a)}$?

T7) A *cuboctahedron*, shown below, is a polyhedron with 8 equilateral triangle faces and 6 square faces. Each edge has the same length and each of the 24 vertices borders 2 squares and 2 triangles. An *octahedron* is a regular polyhedron with 6 vertices and 8 equilateral triangle faces. Compute the sum of the volumes of an octahedron with side length 5 and a cuboctahedron with side length 5.



T8) As a, b and c range over *all* real numbers, let m be the smallest possible value of

$$2\left(a+b+c\right)^2+\left(ab-4\right)^2+\left(bc-4\right)^2+\left(ca-4\right)^2$$

and n be the number of ordered triplets (a, b, c) such that the above quantity is minimized. Compute $m+n$.

T9) Let ABC be a triangle with $AB = 8$, $BC = 9$, and $CA = 10$. The line tangent to the circumcircle of ABC at A intersects the line BC at T, and the circle centered at T passing through A intersects the line AC for a second time at S. If the angle bisector of $\angle SBA$ intersects SA at P, compute the length of segment SP.

T10) Plusses and minuses are inserted in the expression

$$\pm 1 \pm 2 \pm 3 \ldots \pm 2016$$

such that when evaluated the result is divisible by 2017. Let there be N ways for this to occur. Compute the remainder when N is divided by 503.