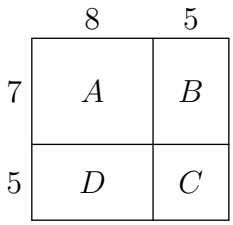
Team Round

CCA Math Bonanza

20 Jan 2018

T1) In the diagram of rectangles below, with lengths as labeled, let A be the area of the rectangle labeled A, and so on. Find $36A + 6B + C + 6D$.



T2) Arnold has plates weighing 5, 15, 25, 35, or 45 pounds. He lifts a barbell, which consists of a 45-pound bar and any number of plates that he has. Vlad looks at Arnold's bar and is impressed to see him bench-press 600 pounds. Unfortunately, Vlad mistook each plate on Arnold's bar for the plate one size heavier, and Arnold was actually lifting 470 pounds. How many plates did Arnold have on the bar?

T3) In the game of Avalon, there are 10 players, 4 of which are bad. A *quest* is a subset of those players. A quest fails if it contains at least one bad player. A randomly chosen quest of 3 players happens to fail. What is the probability that there is exactly one bad player in the failed quest?

T4) $ABCD$ is a convex quadrilateral with $AB = 36$, $CD = 9$, $DA = 39$, and $BD = 15$. Given that $\angle C$ is right, compute the area of $ABCD$.

T5) Call a day a *perfect* day if the sum of the digits of the month plus sum of the digits of the day equals the sum of digits of the year. For example, February 28th, 2028 is a perfect day because $2 + 2 + 8 = 2 + 0 + 2 + 8$. Find the number of perfect days in 2018.

T6) Circle Γ with radius 1 is centered at point A on the circumference of circle ω with radius 7. Suppose that point P lies on ω with $AP = 4$. Determine the product of the distances from P to the two intersections of ω and Γ.

T7) Compute

$$\sum_{i=0}^{\frac{q-1}{2}} \left\lfloor \frac{ip}{q} \right\rfloor + \sum_{j=0}^{\frac{p-1}{2}} \left\lfloor \frac{jq}{p} \right\rfloor$$

if $p = 51$ and $q = 81$.

T8) A rectangular prism with positive integer side lengths formed by stacking unit cubes is called *bipartisan* if the same number of unit cubes can be seen on the surface as those which cannot be seen on the surface. How many non-congruent bipartisan rectangular prisms are there?

T9) 21 Savage has a 12 car garage, with a row of spaces numbered $1, 2, 3, \ldots, 12$. How many ways can he choose 6 of them to park his 6 identical cars in, if no 3 spaces with consecutive numbers may be all occupied?

T10) The irrational number $\alpha > 1$ satisfies $\alpha^2 - 3\alpha - 1 = 0$. Given that there is a fraction $\frac{m}{n}$ such that $n < 500$ and $\left|\alpha - \frac{m}{n}\right| < 3 \cdot 10^{-6}$, find m.