

Tiebreaker Round

CCA Math Bonanza

20 Jan 2018

TB1) What is the maximum number of diagonals of a regular 12-gon which can be selected such that no two of the chosen diagonals are perpendicular?

Note: sides are not diagonals and diagonals which intersect outside the 12-gon at right angles are still considered perpendicular.

TB2) Define a sequence of polynomials $P_0(x) = x$ and $P_k(x) = P_{k-1}(x)^2 - (-1)^k k$ for each $k \geq 1$. Also define $Q_0(x) = x$ and $Q_k(x) = Q_{k-1}(x)^2 + (-1)^k k$ for each $k \geq 1$. Compute the product of the distinct real roots of

$$P_1(x) Q_1(x) P_2(x) Q_2(x) \cdots P_{2018}(x) Q_{2018}(x).$$

TB3) Given that 5^{2018} has 1411 digits and starts with 3 (the leftmost non-zero digit is 3), for how many integers $1 \leq n \leq 2017$ does 5^n start with 1?

TB4) Triangle ABC is a triangle with side lengths 13, 14, and 15. A point Q is chosen uniformly at random in the interior of $\triangle ABC$. Choose a random ray (of uniformly random direction) with endpoint Q and let it intersect the perimeter of $\triangle ABC$ at P . What is the expected value of QP^2 ?