Lightning Round

CCA Math Bonanza

2 Feb 2019

Set 1

Each question in this set is worth 1.5 points.

L1.1) How many integers divide either 2018 or 2019? Note: 673 and 1009 are both prime.

L1.2) At Kanye Crest Academy, employees get paid in CCA Math Bananas™. At the end of 2018, Professor Shian Bray was given a 10% pay raise from his salary at the end of 2017. However, inflation caused the worth of a CCA Math Banana™ to decrease by 1%. If Prof. Bray's salary at the end of 2017 was worth one million dollars, how much (in dollars) was Prof. Bray's salary worth at the end of 2018? Assume that the value of the dollar has not changed.

L1.3) Points P and Q are chosen on diagonal AC of square $ABCD$ such that $AB = AP = CQ = 1$. What is the measure of $\angle PBQ$ in degrees?

L1.4) What is the smallest prime number p such that $1 + p + p^2 + \ldots + p^{p-1}$ is *not* prime?

Set 2

Each question in this set is worth 1.75 points.

L2.1) Noew is writing a 15-problem mock AIME consisting of four subjects of problems: algebra, geometry, combinatorics, and number theory. The AIME is considered *somewhat evenly distributed* if there is at least one problem of each subject and there are at least six combinatorics problems. Two AIMEs are considered *similar* if they have the same subject distribution (same number of each subject). How many non-similar somewhat evenly distributed mock AIMEs can Noew write?

L2.2) What is the largest positive integer n for which there are no *positive* integers a, b such that $8a + 11b = n$?

L2.3) Compute $\sin^4(7.5^\circ) + \sin^4(82.5^\circ)$.

L2.4) Let $ABCD$ be a parallelogram. Let G, H be the feet of the altitudes from A to CD and BC respectively. If $AD = 15$, $AG = 12$, and $AH = 16$, find the length of AB.

Set 3

Each question in this set is worth 2.25 points.

L3.1) Suppose that N is a three digit number divisible by 7 such that upon removing its middle digit, the remaining two digit number is also divisible by 7. What is the minimum possible value of N?

L3.2) What is the area of a triangle with side lengths 17, 25, and 26?

L3.3) 64 teams with distinct skill levels participate in a knockout tournament. In each of the 6 rounds, teams are paired into match-ups and compete; the winning team moves on to the next round and the losing team is eliminated. After the second-to-last round, winners compete for first and second and losers compete for third and fourth. Assume that the team with higher skill level always wins. What is the probability that the first, second, and third place teams have the highest, second highest, and third highest skill levels, respectively?

L3.4) Determine the maximum possible value of

$$\frac{(x^2+5x+12)(x^2+5x-12)(x^2-5x+12)(-x^2+5x+12)}{x^4}$$

over all non-zero real numbers x.

Set 4

Each question in this set is worth 2.5 points.

L4.1) The Garfield Super Winners play 100 games of foosball, in which teams score a non-negative integer number of points and the team with more points after ten minutes wins (if both teams have the same number of points, it is a draw). Suppose that the Garfield Super Winners score an average of 7 points per game but allow an average of 8 points per game. Given that the Garfield Super Winners never won or lost by more than 10, what is the largest possible number of games that they could win?

L4.2) GM Bisain's IQ is so high that he can move around in 10 dimensional space. He starts at the origin and moves in a straight line away from the origin, stopping after 3 units. How many lattice points can he land on? A lattice point is one with all integer coordinates.

L4.3) Let ABC be a triangle with area K. Points A^*, B^*, and C^* are chosen on AB, BC, and CA respectively such that $\triangle A^*B^*C^*$ has area J. Suppose that

$$\frac{AA^*}{AB}=\frac{BB^*}{BC}=\frac{CC^*}{CA}=\frac{J}{K}=x$$

for some $0<x<1$. What is x?

L4.4) If an angle $0^\circ<\theta<30^\circ$ satisfies $\sin(90^\circ-\theta)\sin(60^\circ-\theta)\sin(30^\circ-\theta)=\sin^3(\theta)$, compute $\sin(\theta)$.

Set 5

Each question in this set is worth 2 points.

L5.1) Let $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for any integer $n \geq 3$. For some integer $k > 1$, Johnny converts F_k kilometers to miles, then rounds to the nearest integer. Assume that 1 mile is exactly 1.609344 kilometers. Estimate the smallest value of k such that Johnny *does not* get that this is F_{k-1} miles. An estimate of E earns $2^{1-|A-E|}$ points, where A is the actual answer.

L5.2) Suppose that a planet contains $(CCAMATHBONANZA_{71})^{100}$ people (100 in decimal), where in base 71 the digits $A, B, C, \ldots, Z$ represent the decimal numbers $10, 11, 12, \ldots, 35$, respectively. Suppose that one person on this planet is snapping, and each time they snap, at least half of the current population disappears. Estimate the largest number of times that this person can snap without disappearing. An estimate of E earns $2^{1-\frac{1}{200}|A-E|}$ points, where A is the actual answer.

L5.3) For a positive integer n, let $d(n)$ be the number of positive divisors of n (for example $d(39) = 4$). Estimate the average value that $d(n)$ takes on as n ranges from 1 to 2019. An estimate of E earns $2^{1-|A-E|}$ points, where A is the actual answer.

L5.4) Submit an integer between 0 and 100 inclusive as your answer to this problem. Suppose that Q_1 and Q_3 are the medians of the smallest 50% and largest 50% of submissions for this question. Your goal is to have your submission close to $D = Q_3 - Q_1$. If you submit N, your score will be $2 - 2\sqrt{\frac{|N-D|}{\max\{D,100-D\}}}$.