Team Round

CCA Math Bonanza

2 Feb 2019

T1) Will has a sock drawer with 2 socks of each color: red, green, blue, white, black (socks of the same color are indistinguishable). He absentmindedly grabs 2 socks out of the drawer. What is the probability that he got a pair of matching socks?

T2) A triangle has side lengths of $x, 75, 100$ where $x < 75$ and altitudes of lengths $y, 28, 60$ where $y < 28$. What is the value of $x + y$?

T3) What is the sum of all possible values of $\cos(2\theta)$ if $\cos(2\theta) = 2\cos(\theta)$ for a real number θ?

T4) Find the number of ordered tuples (C, A, M, B) of non-negative integers such that

$$C! + C! + A! + M! = B!$$

T5) What is the smallest positive integer n such that there exists a choice of signs for which

$$1^2 \pm 2^2 \pm 3^2 \ldots \pm n^2 = 0$$

is true?

T6) Compute $\displaystyle\sum_{n=3}^{\infty} \frac{n^2 - 2}{(n^2 - 1)(n^2 - 4)}$.

T7) How many ordered triples (a, b, c) of positive integers are there such that at least two of a, b, c are prime and $abc = 11(a + b + c)$?

T8) fantasticbobob is proctoring a room for the SiSiEyMB with 841 seats arranged in 29 rows and 29 columns. The contestants sit down, take part 1 of the contest, go outside for a break, and come back to take part 2 of the contest. fantasticbobob sits among the contestants during part 1, also goes outside during break, but when he returns, he finds that his seat has been taken. Furthermore, each of the 840 contestants now sit in a chair horizontally or vertically adjacent to their original chair. How many seats could fantasticbobob have started in?

T9) Points P, Q, and M lie on a circle ω such that M is the midpoint of minor arc PQ and $MP = MQ = 3$. Point X varies on major arc PQ, MX meets segment PQ at R, the line through R perpendicular to MX meets minor arc PQ at S, MS meets line PQ at T. If $TX = 5$ when MS is minimized, what is the minimum value of MS?

T10) Define three sequences a_n, b_n, c_n as $a_0 = b_0 = c_0 = 1$ and

$$a_{n+1} = a_n + 3b_n + 3c_n$$
$$b_{n+1} = a_n + b_n + 3c_n$$
$$c_{n+1} = a_n + b_n + c_n$$

for $n \geq 0$. Let A, B, C be the remainders when $a_{13^4}, b_{13^4}, c_{13^4}$ are divided by 13. Find the ordered triple (A, B, C).