Tiebreaker Round

CCA Math Bonanza

2 Feb 2019

TB1) Compute $1^4 + 2^4 + 3^4 + 4^4 + 5^4 + 6^4$.

TB2) Isosceles triangle $\triangle ABC$ has $\angle ABC = \angle ACB = 72^\circ$ and $BC = 1$. If the angle bisector of $\angle ABC$ meets AC at D, what is the positive difference between the perimeters of $\triangle ABD$ and $\triangle BCD$?

TB3) For $i = 1, 2, \ldots, 7$, Zadam Heng chooses a positive integer m_i at random such that each positive integer k is chosen with probability $\frac{2^i-1}{2^{ik}}$. If $m_1 + 2m_2 + \ldots + 7m_7 \neq 35$, Zadam keeps rechoosing the m_i until this equality holds. Given that he eventually stops, what is the probability that $m_4 = 1$ when Zadam stops?

TB4) The number 28! (28 in decimal) has base 30 representation

$$28! = Q6T32S??OCLQJ6000000_{30}$$

where the seventh and eighth digits are missing. What are the missing digits? In base 30, we have that the digits $A = 10$, $B = 11$, $C = 12$, $D = 13$, $E = 14$, $F = 15$, $G = 16$, $H = 17$, $I = 18$, $J = 19$, $K = 20$, $L = 21$, $M = 22$, $N = 23$, $O = 24$, $P = 25$, $Q = 26$, $R = 27$, $S = 28$, $T = 29$.