Team Round

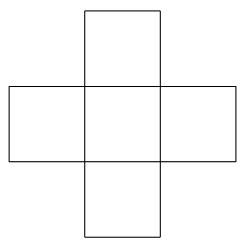
CCA Math Bonanza

18 Jan 2020

T1) Compute the number of permutations of $\{1, 2, 3\}$ with the property that there is some number that can be removed such that the remaining numbers are in increasing order. For example, $(2, 1, 3)$ has this property because removing 1 leaves $(2, 3)$, which is in increasing order.

T2) The base 4 repeating decimal $0.\overline{12}_4$ can be expressed in the form $\frac{a}{b}$ in base 10, where a and b are relatively prime positive integers. Compute the sum of a and b.

T3) Five unit squares are arranged in a plus shape as shown:



What is the area of the smallest circle containing the interior and boundary of the plus shape?

T4) Compute

$$\left(\frac{4 - \log_{36} 4 - \log_6 18}{\log_4 3}\right) \cdot \left(\log_8 27 + \log_2 9\right).$$

T5) Find all pairs of real numbers (x, y) satisfying both equations

$$3x^2 + 3xy + 2y^2 = 2$$
$$x^2 + 2xy + 2y^2 = 1.$$

T6) A cat can see 1 mile in any direction. The cat walks around the 13 mile perimeter of a triangle. Over the course of its walk, it sees every point inside of this triangle. What is the largest possible area, in square miles, of the total region it sees?

T7) Compute the remainder when 99989796 . . . 121110090807 . . . 01 is divided by 010203 . . . 091011 . . . 9798 (note that the first one starts at 99, and the second one ends at 98).

T8) Call an *ordered* triple (a, b, c) *d-tall* if there exists a triangle with side lengths a, b, c and the height to the side with length a is d. Suppose that for some positive integer k, there are exactly 210 k-tall ordered triples of positive integers. How many k-tall ordered triples (a, b, c) are there such that a triangle ABC with $BC = a, CA = b, AB = c$ satisfies both $\angle B < 90°$ and $\angle C < 90°$?

T9) A game works as follows: the player pays 2 tokens to enter the game. Then, a fair coin is flipped. If the coin lands on heads, they receive 3 tokens; if the coin lands on tails, they receive nothing. A player starts with 2 tokens and keeps playing this game until they do not have enough tokens to play again. What is the expected value of the number of tokens they have left at the end?

T10) In $\triangle ABC$ with an obtuse angle at A, let D be the foot of the A altitude and E be the foot of the B altitude. If $AC + CD = DB$ and $BC - AE = EC$, compute $\angle A$ in degrees.