Tiebreaker Round

CCA Math Bonanza

18 Jan 2020

TB1) In a group of 2020 people, some pairs of people are friends (friendship is mutual). It is known that no two people (not necessarily friends) share a friend. What is the maximum number of unordered pairs of people who are friends?

TB2) Shayan is playing a game by himself. He picks **relatively prime** integers a and b such that $1 < a < b < 2020$. He wins if every integer $m \geq \frac{ab}{2}$ can be expressed in the form $ax + by$ for nonnegative integers x and y. He hasn't been winning often, so he decides to write down all winning pairs (a, b), from (a_1, b_1) to (a_n, b_n). What is $b_1 + b_2 + \ldots + b_n$?

TB3) How many unordered triples A, B, C of distinct lattice points in $0 \leq x, y \leq 4$ have the property that $2[ABC]$ is an integer divisible by 5?

Note: $[ABC]$ denotes the area of $\triangle ABC$. It is 0 whenever A, B, and C are collinear.

TB4) Let ABC be a triangle with $AB = 13$, $BC = 14$, and $CA = 15$. The incircle of ABC meets BC at D. Line AD meets the circle through B, D, and the reflection of C over AD at a point $P \neq D$. Compute AP.