Tiebreaker Round

CCA Math Bonanza

17 Apr 2021

TB1) Consider the set of all ordered 6-tuples of nonnegative integers (a, b, c, d, e, f) such that

$$a + 2b + 6c + 30d + 210e + 2310f = 2^{15}.$$

In the tuple with the property that $a+b+c+d+e+f$ is minimized, what is the value of c?

TB2) Convex quadrilateral $ABCD$ with perpendicular diagonals satisfies $\angle B = \angle C = 90^\circ$, $BC = 20$, and $AD = 30$. Compute the square of the area of a triangle with side lengths equal to CD, DA, and AB.

TB3) In a party of 2020 people, some pairs of people are friends. We say that a given person's *popularity* is the size of the largest group of people in the party containing them with the property that every pair of people in that group is friends. A person has popularity number 1 if they have no friends. What is the largest possible number of distinct popularities in the party?

TB4) For all integers $0 \leq k \leq 16$, let

$$S_k = \sum_{j=0}^{k} (-1)^j \binom{16}{j}.$$

Compute $\max(S_0, S_1, \ldots S_{16})$.