Lightning Round

CCA Math Bonanza

23 Apr 2022

Set 1

Each question in this set is worth 1.5 *points.*

L1.1) Given

$$a = bc$$
$$b = ca$$
$$c = a + b$$
$$c > a$$

Evaluate $a + b + c$.

L1.2) Xonathan Jue goes to the casino with exactly \$1000. Each week, he has a 1/3 chance of breaking even and 2/3 chance of losing \$500. Evaluate the expected amount of weeks before he loses everything.

L1.3) The area of the region bound by the equations $y = 2\cos(4x)$, $y = \sin(2x) + 10$, $x = 0$, and $x = 5\pi$ can be expressed as $x\pi$ for some value of x. Evaluate x.

L1.4) Jongol and Gongol are writing calculus questions and grading tests. They want to write 90 calculus problems and they have 120 tests to grade. Jongol can write 3 questions per minute or grade 4 tests per minute. Gongol can write 1 question per minute or grade 2 tests per minute. Evaluate the shortest possible time, in minutes, for them to complete the two tasks.

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Set 2

Each question in this set is worth 1.75 *points.*

L2.1) Given that a duck found that $5 - 2\sqrt{3}i$ is one of the roots of $-259 + 107x - 17x^2 + x^3$, what is the sum of the real parts of the other two roots?

L2.2) A rectangle $ABCD$ has side lengths $AB = 6$ miles and $BC = 9$ miles. A pigeon hovers at point P, which is 5 miles above some randomly chosen point inside $ABCD$. Given that the expected value of

$$AP^2 + CP^2 - BP^2 - DP^2$$

can be expressed as $\frac{a}{b}$, what is ab?

L2.3) Given that the height of a greater sage grouse flying through the air is defined by the function $64x - x^2$ for $0 < x < 64$, what is the first time at which the bird reaches a height of 903?

L2.4) 10 geese are numbered 1-10. One goose leaves the pack, and the remaining nine geese assemble in a symmetric V-shaped formation with four geese on each side. Given that the product of the geese on both halves of the "V" are the same, what is the sum of the possible values of the goose that left?
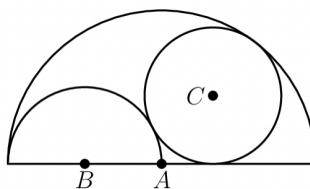
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Set 3

Each question in this set is worth 2.25 *points.*

L3.1) Kongol rolls two fair 6-sided die. The probability that one roll is a divisor of the other can be expressed as $\frac{p}{q}$. Determine $p + q$.

L3.2) In the following diagram, $AB = 1$. The radius of the circle with center C can be expressed as $\frac{p}{q}$. Determine $p + q$.

L3.3) Determine the sum of all positive integers $n < 100$ satisfying the following expression.

$$\sum_{k=0}^{\lfloor \log_{10} n \rfloor} \frac{1}{10^k} \left(n \ (\mathrm{mod} \ 10^{k+1}) - n \ (\mathrm{mod} \ 10^k) \right) = \prod_{k=0}^{\lfloor \log_{10} n \rfloor} \frac{1}{10^k} \left(n \ (\mathrm{mod} \ 10^{k+1}) - n \ (\mathrm{mod} \ 10^k) \right).$$

Here, $\sum$ and $\prod$ represent sum and product, respectively.

L3.4) A merganser mates every 7th day, a scaup mates every 11th day, and a gadwall mates every 13th day. A merganser, scaup, and gadwall all mate on Day 0. On Days N, N+1, and N+2 the merganser, scaup, and gadwall mate in some order with no two birds mating on the same day. Determine the smallest possible value of N.

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Set 4

Each question in this set is worth 2.5 *points.*

L4.1) There are 99 seagulls labeled 2-100 and 100 bagels labeled 1-100. Starting from Seagull 2, each Seagull N eats $\frac{1}{N}$ of whatever remains of each Bagel I where N divides I. How many bagels still have more than $\frac{2}{3}$ of their original size after Seagull 100 finishes eating?

L4.2) A rhombicosidodecahedron is an Archimedean solid, one of thirteen convex isogonal nonprismatic solids constructed of two or more types of regular polygon faces. It has 20 regular triangular faces, 30 square faces, and 12 regular pentagonal faces, as shown below.
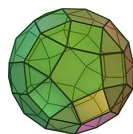
How many rotational symmetries does a rhombicosidodecahedron have?

L4.3) Ethan Song and Bryan Guo are playing an unfair game of rock-paper-scissors. In any game, Ethan has a 2/5 chance to win, 2/5 chance to tie, and 1/5 chance to lose. How many games is Ethan expected to win before losing?

L4.4) Lukas Nepomuceno draws 5 congruent circles equally spaced around a 6th, and colors all of them 1 of 3 colors. Assume that rotations and reflections of colorings are indistinguishable. How many distinct colorings are there?

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Set 5

Each question in this set is worth 2 *points. Please submit a positive integer or decimal for each question.*

L5.1) Alistar wants to wreak havoc on Jhin's yard, which is a 2D plane of grass. First, he selects a number n, randomly and uniformly from $[0, 1]$, and then he eats all grass within n meters from where he's standing. He then moves 2 meters in a random direction, and repeats his process. He stops if any of the grass that he wants to eat (or, in other words, in his intended eating territory) is already eaten. Estimate the amount of grass Alistar is expected to eat. An estimate E earns $\frac{2}{1+|A-E|}$ points, where A is the actual answer.

L5.2) Estimate the range of the submissions for this problem. Your answer must be between $[0, 1000]$. An estimate E earns $\frac{2}{1+0.05|A-E|}$ points, where A is the actual answer.

L5.3) Estimate the number of times a one-digit answer (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9) has been submitted as an answer for any question by any team in the first 4 sets of this competition's lightning round. An estimate E earns $\frac{2}{1+|log_2(A)-log_2(E)|}$ points, where A is the actual answer.

L5.4) Five points are selected within a unit circle at random. Estimate the minimum distance between any pair of points. An estimate E earns $\frac{2}{1+|log_2(A)-log_2(E)|}$ points, where A is the actual answer.